Optimal Quantization and Energy Allocation Schemes in Distributed Estimation for Wireless Sensor Networks

Eni Dwi Wardihani, Wirawan, and Gamantyo Hendrantoro
Dept. Of Electrical Engineering, Institut Teknologi Sepuluh Nopember Surabaya, Indonesia

E-mail: wardihani09@mhs.ee.its.ac.id, wirawan@its.ac.id, gamantyo@ee.its.ac.id

Abstract— We consider a distributed estimation for wireless sensor networks wireless sensor network with a fusion center (FC) which collects data from sensor nodes. The sensor quantized their observations and transmitted the quantized observations to a FC over flat fading Rayleigh channel with path-loss effects. We consider two allocation strategies as the goals of this paper. These are the optimal energy allocation scheme and the optimal number of quantization bits at sensor nodes under strict energy constraints, while the noisy channel between each sensor and FC modelled as flat-fading Rayleigh channel. Optimality here is defined in the sense of minimizing the reconstruction error at the FC.

I. INTRODUCTION

Recently, wireless sensor networks (WSNs) have attracted much attention and interest, and have become a very active research area. Due to their high flexibility, enhanced surveillance coverage, robustness, mobility, and cost effectiveness, WSNs have wide applications and high potential in military surveillance, security, monitoring of traffic, and environment. Usually, a WSN consists of a large number of low cost and low-power sensors, which are deployed in the environment to collect observations and preprocess the observations. Each sensor node has limited communication capability that allows it to communicate with other sensor nodes via a wireless channel. Normally, there is a fusion center that processes data from sensors and forms a global situational assessment [1].

In most application scenarios, WSN nodes are powered by small batteries, which are practically non-rechargeable, either due to cost limitations or because they are deployed in hostile environments with high temperature, high pollution levels, or high nuclear radiation levels. These considerations motivate energy-saving and energy-efficient WSN designs [2]. Various energy-efficient algorithms have been proposed for network coverage, data gathering, and protocols of medium access control and routing. These references focus on collaborative strategies and cross-layer designs for distributed data collection, processing, and communication in an energy-efficient manner [3].

In this paper, we consider a distributed estimation for wireless sensor networks wireless sensor network with a fusion center which collects data from sensor nodes. Each sensor quantizes their observations and transmits the quantized observations to a FC, where source parameter is estimated. We model the wireless links between sensors and the FC as a flat-fading Rayleigh channel with path-loss effects. The goal of this paper is to derive optimal number of quantization bits and optimization of the energy allocation schemes at sensor nodes under strict energy constraints. Optimality here is defined in the sense of minimizing the estimation error at the fusion center.

Most of the prior works on optimal quantization deal with optimization of the quantization rules for detecting a signal in dependent or independent noise [4-7]. Optimization of the quantization per node under a fixed total energy per measurement in order to minimize the reconstruction error at the FC when the channel between each sensor and the fusion center is modeled as a BSC designed in [8]. Different from these works, our objective is to optimize the energy allocation per sensor under a given energy budget so that this energy allocation scheme achieve the minimum possible reconstruction error. In this paper we consider the wireless channel between the sensor and the fusion center as a flat-fading Rayleigh channel with path-loss effects.

The rest of the paper is organized as follows. Section 2, discusses the problem formulation. We consider a overview of decentralized estimation schemes for WSN in section 3. In section 4, optimal energy allocation scheme with identical number of bits per sensor. Section 5 provides the energy allocation schemes for different number of bits per sensor. We provides some illustrative numerical results in section 6 and section 7 concludes the paper.

II. PROBLEM FORMULATION

Let us consider the system depicted in Fig. 1. A set of $K$ distributed sensors, each making observations on deterministic source signal $\theta$. The observations are corrupted by additive noises and are described by:

$$X_k = \theta + n_k \quad k = 1, ..., K$$ (1)

The noise $\{n_k : k = 1, ..., K\}$ are zero mean spatially uncorrelated with variance $\sigma^2$, and independent of $n_l$ for $l \neq k$. Suppose sensors and the fusion center wish to jointly estimate $\theta$ based on the sensors observation $\{X_k\}$. We will use MSE to measure the quality of an estimator. If the fusion center has the knowledge of sensor noise variances and the sensors can send the observations $X_k : k = 1, ..., K$ to the fusion center without distortion, then the fusion center can simply perform
the linear combination of sensor observations to recover \( \theta \). The solution based on centralized BLUE criterion is given [9]:

\[
\tilde{\theta}_k(X_1, X_2, \ldots, X_K) = \left( \sum_{k=1}^{K} \frac{1}{\sigma_k^2} \right)^{-1} \sum_{k=1}^{K} \frac{X_k}{\sigma_k^2}
\]

(2)

The above scheme is impractical for implementation in sensor networks because sensor observations can never be perfectly transmitted to the FC due the finite data rate and energy constraints of the communication channels in a WSN. In this paper, we propose the following decentralized estimation scheme, Fig.1. Each sensor performs a local quantization of \( x_i \) and generates a quantization message \( m_i(x_i, N_i) \) of \( N_i \) bits. The modulated bits then transmitted through wireless channel, which is modeled as a flat-fading Rayleigh channels with path-loss effects.

We investigate the optimal scheme for allocating the total energy prescribed to all sensors so that the error reconstruction at fusion center minimized. We derive the optimal number of quantization bits per sensor so that energy allocation scheme achieves the minimum possible reconstruction errors.

III. DECENTRALIZED ESTIMATION

A set of \( K \) distributed sensors, each making observations. After the normalization, we have \( X_k \in [0,1] \). Each sensor performs a local quantization of \( X_k \) and generates a quantization message \( m_i(X_k, N_i) \) of \( N_i \) bits, with \( X_k = \sum_{i=1}^{N_i} b_i^{(k)} 2^{-i} \), we have \( m_k = \sum_{i=1}^{N_k} b_i^{(k)} 2^{-i} \). Bits \( \{b_i^{(k)}\}_{i=1}^{N_k} \) are then transmitted through wireless channel, which is modeled as a flat-fading Rayleigh channels with path-loss effects. The FC reconstructs \( X_k \) with the demodulated bits \( \{b_i^{(k)}\}_{i=1}^{N_k} \) to obtain:

\[
\hat{X}_k = \sum_{i=1}^{N_k} \hat{b}_i^{(k)} 2^{-i}
\]

(3)

The estimation error in FC is given:

\[
\tilde{\theta} - \theta = \left( \sum_{k=1}^{K} \frac{1}{\sigma_k^2} \right)^{-1} \sum_{k=1}^{K} \frac{\hat{X}_k - \theta}{\sigma_k^2}
\]

\[
= \left( \sum_{k=1}^{K} \frac{1}{\sigma_k^2} \right)^{-1} \sum_{k=1}^{K} \frac{\hat{X}_k - X_k + n_k}{\sigma_k^2}
\]

(4)

Upon defining the reconstruction error \( \tilde{X}_k = \hat{X}_k - X_k \) we have

\[
E|\tilde{\theta} - \theta|^2 = \left( \sum_{k=1}^{K} \frac{1}{\sigma_k^2} \right)^{-1} \sum_{k=1}^{K} \frac{E|\tilde{X}_k + n_k|^2}{\sigma_k^2}
\]

\[
= E\left[\frac{K}{\sigma_k^2} + \frac{1}{\sigma_k^2} (\tilde{X}_k)^2 + \frac{K}{\sigma_k^2} (n_k)^2 \right]
\]

(5)

As long as the quantization step size \( \Delta = 2^{-N_k} \) is sufficiently small relative to \( \sigma_k \), we can assume that the reconstruction error \( \tilde{X}_k = X_k - m_k + m_k - X_k \) is statistically uncorrelated with the observation noise \( n_k \) [10]. Thus, the second summand in the numerator disappears. Hence, minimizing \( E|\tilde{\theta} - \theta|^2 \) reduces to minimizing

\[
E\left[\sum_{k=1}^{K} \frac{X_k + n_k}{\sigma_k^2} \right]^2
\]

(6)

where \( \epsilon_k \) is the probability error of the flat-fading Rayleigh channel between sensor and FC.

IV. IDENTICAL NUMBER OF BITS PER SENSOR

For clarity in exposition, we first consider here a simple situation where each sensor transmits the same fixed number of bits \( N \) (\( N_k = N, \forall k \)). With \( x_k \) denoting the fraction of the total energy \( \epsilon_T \) allocated to sensor \( k \), we can express \( \epsilon \) as \( \epsilon(\epsilon_T x_k/NN_0) \). Notice that the noise level at the receiver of the fusion center is assumed common to all channels. The optimal energy allocation scheme will be the solution of the following optimization problem \( (x := [x_1, \ldots, x_K]^T) \):

\[
\text{minimize}_{x} f_0(x; N) := \sum_{k=1}^{K} \frac{1}{2 \nu^2} + \left( 1 - \frac{1}{2 \nu} \right) \epsilon_k \left( \frac{\epsilon_T x_k}{NN_0} \right)
\]

subject to \( f_k(x) := -x_k \leq 0, \quad k = 1, \ldots, K, \)

\[
h(x) := \sum_{k=1}^{K} x_k = 1
\]

We can write down the KKT conditions [13] for the optimal solutions \( (x := [x_1, \ldots, x_K]^T) \) as follows:

\[
x_k^* \geq 0, \quad \lambda_k^* \geq 0, \quad \lambda_k^* x_k^* = 0, \quad k = 1, 2, \ldots, K
\]

(8)

\[
\sum_{k=1}^{K} x_k^* = 1
\]

(9)

\[
\forall f_0(x^*; N) + \sum_{k=1}^{K} \lambda_k^* \nabla f_k(x^*) + \nu' \nabla h(x_k^*) = 0
\]

(10)

where \( \nabla \) denotes the gradient. It follows from (9) that the must satisfy

\[
\frac{1}{\epsilon_T} \frac{d \epsilon(x; N)}{d x_k} \bigg|_{x_k = \epsilon_T NN_0 x_k} = -\lambda_k + \nu' = 0, \quad k = 1, \ldots, K
\]

(11)
We consider the channel between the sensor and the FC experiences a path loss proportional to $d_k^\alpha$, where $d_k$ is the distance between sensor $k$ and the FC and $\alpha$ is path loss exponent [11]. Using BPSK modulation, we can express the probability that bit $\gamma$ is given by $e(\gamma) = 1/4 \left( \frac{\gamma}{d_k^\alpha} \right) = \frac{d_k^\alpha}{4\gamma}[12]$. Under these operating conditions (8) and (11) yield

$$x_k^* = \phi^{-1} \left( \frac{e\gamma}{\phi(\gamma)} \right) \left( \frac{NN_0}{\phi(\gamma)} \right)$$

$$\phi(\gamma) = \frac{d_k^\alpha}{4\gamma}$$

where $\nu^*$ is chosen such that $\sum_{k=1}^K x_k^* = 1$. In Section 6, we will examine a specific system and find the corresponding optimal energy allocation to gain further insight into these closed-form expressions.

In fact, when $e_k(\gamma)$ for all $k$ is convex in $\gamma$, the problem in (7) turns out to be convex, which implies that the global optimum exists and can be easily found numerically. In most cases, convexity is guaranteed. The optimal number of quantization bits $N_{opt}$ can be easily found using one-dimensional numerical search to solve the optimization problem:

$$N_{opt} = \arg \min_N f_0(x^*;N)$$

Where $f_0(x^*;N)$ is the optimal value of the objective function in (7) when the number of quantization bits per sensor is $N$. In Section 5, we will show an example of the functional relationship between $f_0(x^*;N)$ and $N$ from which $N_{opt}$ can be readily determined.

V. DIFFERENT NUMBER OF BITS PER SENSOR

We consider the case where the $k^{th}$ sensor transmits $N_k$ quantization bits, $k=1,...,K$. The optimal energy allocation to minimize the estimation error is the solution of the following optimization problem:

$$\text{minimize } f_0(x;N_k)$$

subject to $f_k(x) := -x_k \leq 0, \quad k = 1,...,K$,

$$h(x) := \sum_{k=1}^K x_k = 1$$

Given the set of $N_k$ the solution to (15) is similar to (7). The minimization of $f_0(x;N_k)$ with respect to $N_k, k=1,...,K$ and $x_k, k=1,...,K$ to find the optimal solution $N_k,k=1,...,K$ and $x_k, k=1,...,K$ jointly follows algorithm 1.

When $e_k(\gamma)$ is convex and $\{N_k\}_{k=1}^K$ are fixed, the problem in (15) is clearly convex, which implies that the optimal energy allocation vector $x_k$ can be found using standard numerically efficient search schemes [13] Hence, step (i) of Algorithm 1 is easily carried out. It is also easy to prove that the objective function is always decreasing from one iteration to another. From the simulations show that Algorithm 1 typically converges after 3-4 iterations.

i. at 1st step, with $N_k^{(1)}, k = 1,...,K$ find $x^{(1)} = [x_1^{(1)},...,x_K^{(1)}]^T$ as the optimal solution of (14).
ii. update $N_k^{(l)}$ to $N_k^{(l+1)}$ based on iteration

$$N_k^{(l+1)} = \min_{N_k} \left( \frac{1}{2N_k} + \left( 1 - \frac{1}{2N_k} \right) e_k(\gamma_k^{(l)}) \right)$$

Algorithm 1

VI. NUMERICAL RESULT

In this section, we provide numerical examples to corroborate the analytical results we derived in the previous sections. Suppose that $K=10$ sensors are deployed with local observation noise variances denoted by $\sigma_1^2,\sigma_2^2,...,\sigma_{10}^2$ the path loss exponent of the wireless channel is $\alpha = 2$ (free space), and accordingly, the error probability is given by $e_k(\gamma) = \frac{d_k^\alpha}{4\gamma}$, where $d_k$ is the distance between sensor $k$ and the fusion center. We set the total energy budget to be $\epsilon_T/N_0 = 200$.

A. Identical $N_k = N_k = 1,...,K$

Fig.2 compares the the equal energy allocation and the optimal energy allocation scheme for a variable number of bits $N$ while choosing a specific set of values for $\{d_k\}_{k=1}^{10}$ and $\{\sigma_k^2\}_{k=1}^{10}$. The result show that the optimal energy allocation scheme have better performance than the equal energy allocation scheme. For this particular setup, the optimal value of $N$ in (13) turns out to be $N_{opt} = 6.8$. With different sets of values for $\{d_k\}_{k=1}^{10}$ and $\{\sigma_k^2\}_{k=1}^{10}$, when $N$ is accordingly chosen to be optimal, the corresponding optimal energy allocation schemes are depicted in Fig.3. The optimal energy allocation scheme are influenced by the distance between the sensor and the FC.

B. Different number of bits per sensor $\{N_k, x_k\}_{k=1}^K$

We simulated the different number of quantization bits per sensor as explained in Section 5. We can find the optimal $N_k$ and $x_k, k = 1,...,K$, jointly by utilizing Algorithm 1. Through this algorithm the resulting optimal energy allocation scheme and the optimal number of quantization bits per sensor are depicted in Fig.4 and 5.
VII. CONCLUSION

In this paper we have described the optimal energy allocation scheme and the optimal number of quantization bits per sensor when the channel between sensor and FC modelled as flat-fading Rayleigh channel with path loss effects. We obtained the error reconstruction at the FC can be minimize using optimal energy allocation scheme among sensor. We also show that to minimize error reconstruction at the FC, the optimal energy allocation per sensor depend on the number of quantization bits per sensor, variances noise sensor and distance between sensor and FC. The optimal number of quantization bits per sensor can be found with the help of our convex optimization formulation.

Fig. 2. (a) equal energy allocation among sensors, (b) optimal energy allocation among sensor with $\sigma_2^2 = 0.01/k, k = 1, ..., 10$ and $(d_1, ..., d_{10}) = \{1,5,1,5,1,5,1,5\}$

Fig. 3. Optimal energy allocation scheme for $N_k = N_{opt}, k=1, ..., K$, case 1: $N_{opt} = 6, d_k = k/4$ and $\sigma_2^2 = 0.01 \forall k$, case 2: $N_{opt} = 6, d_k = 1, \forall k$, $\sigma_2^2 = 0.01$, case 3: $N_{opt} = 8, (d_1, ..., d_{10}) = \{1,5,1,5,1,5,1,5\}, \sigma_2^2 = 0.01k$

Fig. 4. Jointly optimized number of quantization bits per sensor, $N_k, k = 1, ..., k$, case 1: $d_k = k/4$ and $\sigma_2^2 = 0.01/k$, case 2: $d_k = 1, \forall k$ and $\sigma_2^2 = 0.01k$, case 3: $(d_1, ..., d_{10}) = \{1,5,1,5,1,5,1,5\}$ and $\sigma_2^2 = 0.01k$

Fig. 5. Jointly optimized energy allocation schemes per sensor, $x_k, k = 1, ..., k$, case 1: $d_k = k/4$ and $\sigma_2^2 = 0.01/k$, case 2: $d_k = 1, \forall k$ and $\sigma_2^2 = 0.01k$, case 3: $(d_1, ..., d_{10}) = \{1,5,1,5,1,5,1,5\}$ and $\sigma_2^2 = 0.01k$

REFERENCES


